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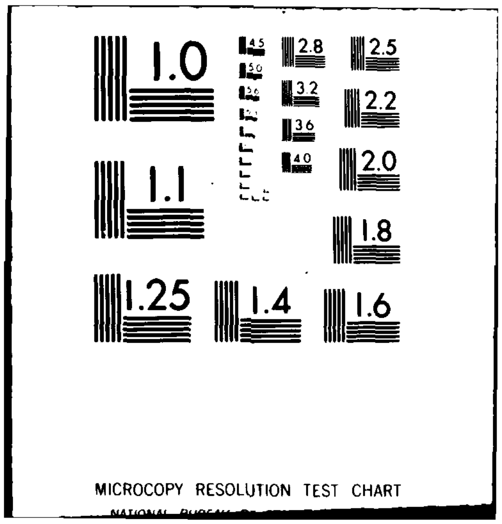
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INSTITUTE FOR PHYSICAL SCIENCE
AND TECHNOLOGY

Laboratory for Numerical Analysis

Technical Note BN-939

ON A DIMENSIONAL REDUCTION METHOD.

THE OPTIMAL SELECTION OF BASIS FUNCTIONS

by

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and

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Laboratory for Numerical Analysis

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Abstract

This paper is the first in a series of three, which analyze an adaptive approximate approach for solving (n+1)- dimensional boundary value problems by replacing them with systems of equations in n-dimensional space. In this approach the unknown functions of (n+1)- variables are projected onto finite linear combinations of functions of just n-variables.

This paper shows how the coefficients of these linear combinations can be chosen optimally.

Introduction

Let $\Omega^h = \omega \times [-h,h]$ be a domain in $\mathbb{R}^{n+1}$, and let u^h be the solution to some elliptic boundary value problem on Ω^h. We wish to find -- in a very effective way -- an approximate solution u^h_{approx} satisfying a certain accuracy requirement.

Considering the special structure of Ω^h, we expect that u^h can be approximated well by a linear combination

$$\sum_{j=0}^{N} \psi_j(y/h)\cdot x_j \ , \quad y \in [-h,h]$$

of $N+1$ functions $\{x_j\}_{j=0}^N$ on ω. Methods built on an assumption of this type and a projection procedure are widely used in engineering. As an example, we mention various theories for plates, beams, etc. (cf. [4,11,12]). These methods are also sometimes associated with the name of L.V. Kantorovich (cf.[10]).

Our goal is to select the family of functions $\{\psi_j\}_{j=0}^{\infty}$ such that

i) The method is optimally accurate when h is small and the data sufficiently regular.

ii) For arbitrary h and input data the method converges as $N \to \infty$.

iii) It is possible to derive an a-posteriori estimate for the error and this leads to an effective procedure for the selection of N.

Another approach, which has also been extensively used in structural mechanics and elsewhere to derive lower dimensional approximating models, is asymptotic expansion in h (cf. [5,8,9]). We refer to [16] and references therein for various engineering applications of this approach. It is quite obvious that a method based on an asymptotic expansion in h does not satisfy the goals stated above. The validity of an approach of this type is dependent on the smallness of h, while the actual value of h may simply not be small enough. But even for arbitrarily small h the approximation can be unsatisfactory because of rough data. [1]

contains an example that shows that even for an extremely thin, simply-supported polygonal plate, the biharmonic equation is not always a good model for a three-dimensional plate.

For the model problem analyzed in this paper we restrict ourself to projection in the energy. We project the solution u^h on elements of the form

$$\sum_{j=0}^{N} \psi_j(y/h)x_j \quad ,$$

where $\{\psi_j\}_{j=0}^{\infty}$ is a sequence of functions on $[-1,1]$. The variable y ranges over $[-h,h]$ and the x_j's are arbitrary elements of some linear space (e.g., a function space on ω) . The main question addressed here is how to select the sequence $\{\psi_j\}_{j=0}^{\infty}$. It has already been stated that one of our goals is to obtain optimal rate of accuracy for small h . In theorems 3.1 and 4.1 we prove that this requirement almost uniquely determines the sequence $\{\psi_j\}_{j=0}^{\infty}$.

In a forthcoming paper [15], we study convergence properties of the method as $N \to \infty$.

The presence of a strong singularity in the data is reflected in the fact that we must use a relatively high number of functions ψ_j (i.e., we must increase N) . Because such singularities are often localized, it seems appropriate to introduce the possibility of using a different N in different parts of the domain Ω^h . The a-posteriori error estimation and the problem of how to design an adaptive algorithm that will produce a good distribution of the N's , was briefly discussed in [14]. It will also be given a more detailed treatment in a forthcoming paper.

2. Notation and the Model Problem

Let H be a separable Hilbert space with inner product $\langle u,v\rangle$ and norm $||u|| = \langle u,u\rangle^{1/2}$.

A denotes a (possibly unbounded) self adjoint linear operator in H with domain of definition $\mathcal{D}(A)$. Furthermore we assume that A is a strictly positive-definite operator, i.e., there exists $C > 0$ such that

$$\forall u \in \mathcal{D}(A): \quad C||u||^2 \leq \langle Au,u\rangle$$

$\mathcal{D}(A^{1/2})$ is itself a Hilbert space with the inner product $\langle u,v\rangle + \langle A^{1/2}u,A^{1/2}v\rangle$.

Let M be a self adjoint bounded linear operator in H . M is also assumed to be a strictly positive-definite operator.

I denotes an interval on the real line. $L^2(I;H)$ is then defined as the set of strongly measurable functions $I \to H$ such that $||u(\cdot)||$ is an element of $L^2(I)$, (cf. [6]). The same goes for $L^2(I;\mathcal{D}(A^{1/2}))$.

We also need a Sobolev space of functions with values in H . $H^1(I;H)$ denotes the space of functions $I \to H$ such that $u(\cdot) \in L^2(I;H)$ and $\frac{d}{dy}u(\cdot) \in L^2(I;H)$, (cf. [2]). The derivative here is taken in the distributional sense. $H^1(I)$ denotes the standard Sobolev space on I .

Assume a and b are real valued functions in $L^\infty([-1,1])$ such that

$$a_o \leq a(y) \; \forall y \in [-1,1] \quad ,$$

$$b_o \leq b(y) \; \forall y \in [-1,1] \quad ,$$

for some constants $a_o > 0$, $b_o > 0$.

a_h and $b_h \in L^\infty([-h,h])$ are then defined as

$$a_h(y) = a(y/h) \qquad \forall y \in [-h,h] \;,$$
$$b_h(y) = b(y/h) \qquad \forall y \in [-h,h] \;.$$

By $P_h(\frac{d}{dy})$ we denote the differential operator $-\frac{d}{dy}(a_h \frac{d}{dy})$.

Let f and g be two arbitrary vectors from H . We consider the following model problem

$$(1) \qquad \left[\begin{array}{rll} P_h(\frac{d}{dy})Mu^h + b_h\, Au^h = 0 & \text{in} &]-h,h[\;, \\ a_h \frac{d}{dy}\, Mu^h = g & \text{for} & y = h \;, \\ a_h \frac{d}{dy}\, Mu^h = f & \text{for} & y = -h \;. \end{array}\right.$$

(Other boundary conditions, e.g., Dirichlet conditions, could just as well have been chosen; we could also consider the inhomogeneous problem. The above selection was simply made for convenience.)

Before we proceed any further, let us give a simple example.

<u>Example</u>

Let ω be a domain in $\mathbb{R}^n$ with a Lipschitz boundary. As H we take $L^2(\omega)$. Let A be the Friedrichs extension (cf. [13]) of the operator $-\mathrm{div}\; c(\underline{x})\;\mathrm{grad}$ defined on a subspace of $\overset{o}{H}{}^1(\omega)$. ($\underline{x}$ denotes coordinates in ω). c is a function in $L^\infty(\omega)$ such that $\exists\, c_o > 0$ with $c_o \le c(\underline{x})\;\; \forall\, \underline{x} \in \omega$.

If we take $a = b$ and let M be the operator of multiplication by $c(\underline{x})$, the problem (1) becomes

$$\begin{cases} \operatorname{div} d_h(\underline{x},y) \operatorname{grad} u^h = 0 \quad \text{in} \quad \omega \times]-h,h[\quad , \\ d_h(\underline{x},y) \frac{\partial}{\partial y} u^h = g(\underline{x}) \quad \text{for} \quad y = h \ , \\ d_h(\underline{x},y) \frac{\partial}{\partial y} u^h = f(\underline{x}) \quad \text{for} \quad y = -h \quad , \\ u^h = 0 \quad \text{on} \quad \partial\omega \times [-h,h] \quad . \end{cases}$$

Here $d_h(\underline{x},y) = b(y/h)\cdot c(\underline{x})$, and div and grad are taken with respect to the $n+1$ coordinates $(\underline{x},y)$.

The precise formulation of (1) is

$$(2) \quad \begin{cases} u^h \in H^1([-h,h];H) \cap L^2([-h,h];\ \mathcal{D}(A^{1/2})) \ , \\ \mathcal{B}_h(u^h,v) = \langle g,v(h)\rangle - \langle f,v(-h)\rangle \quad , \\ \forall v \in H^1([-h,h];H) \cap L^2([-h,h];\ \mathcal{D}(A^{1/2})) \end{cases}$$

where $\mathcal{B}_h$ denotes the bilinear form

$$\mathcal{B}_h(u,v) = \int_{-h}^{h} a_h \langle M^{1/2} \frac{d}{dy} u,\ M^{1/2} \frac{d}{dy} v\rangle \, dy + \int_{-h}^{h} b_h \langle A^{1/2}u,\ a^{1/2}v\rangle \, dy \quad .$$

If $H^1([-h,h];H) \cap L^2([-h,h];\mathcal{D}(A^{1/2}))$ is endowed with the natural norm $|||\cdot|||$:

$$|||u||| = [\int_{-h}^{h} ||\frac{d}{dy} u(y)||^2 dy + \int_{-h}^{h} ||A^{1/2}u(y)||^2 dy]^{1/2} \quad ,$$

then it is not difficult to prove

Proposition 2.1

There exist C_1, C_2 (independent of h) such that

(i) $|\mathcal{B}_h(u,v)| \leq C_1 |||u|||\cdot|||v|||$,

(ii) $|||u|||^2 \leq C_2 \ |\mathcal{B}_h(u,u)|$

$$\forall u,v \in H^1([-h,h];\mathcal{H}) \cap L^2([-h,h];\mathcal{D}(A^{1/2})) \quad .$$

Also one has

Proposition 2.2

If $x \in \mathcal{H}$ and $y_o \in [-h,h]$, then

$$\Lambda : \quad v \to \langle x, v(y_o)\rangle$$

is a continuous linear functional on $H^1([-h,h];\mathcal{H})$.

Proof

From the definition of $H^1([-h,h];\mathcal{H})$ it follows that if $v(\cdot) \in H^1([-h,h];\mathcal{H})$ and λ is a continuous linear functional on $\mathcal{H}$, then

$$\lambda(v(\cdot)) \in H^1([-h,h]) \quad \text{and}$$

$$\left[\int_{-h}^{h} |\lambda(v(y))|^2 dy + \int_{-h}^{h} |\frac{d}{dy}\ \lambda(v(y))|^2 dy\right]^{1/2}$$

$$\leq ||\lambda||' \left[\int_{-h}^{h} ||v(y)||^2 dy + \int_{-h}^{h} ||\frac{d}{dy}\ v(y)||^2 dy\right]^{1/2} \quad ,$$

($||\cdot||'$ is the norm in H') .

Hence, with $\lambda(\cdot) = \langle x,\cdot\rangle$ we get

$$\langle x,v(\cdot)\rangle \in H^1([-h,h]) \quad \text{and}$$

$$\left[\int_{-h}^{h} |\langle x,v(y)\rangle|^2 dy + \int_{-h}^{h} \left|\frac{d}{dy} \langle x,v(y)\rangle\right|^2 dy\right]^{1/2}$$

$$\leq ||x|| \left[\int_{-h}^{h} ||v(y)||^2 dy + \int_{-h}^{h} \left|\left|\frac{d}{dy} v(y)\right|\right|^2 dy\right]^{1/2}$$

Using this last estimate together with the standard trace theorem, we finally get that Λ is a continuous linear functional on $H^1([-h,h];H)$

$\square$

Propositions 2.1 and 2.2 immediately give

<u>Proposition 2.3</u>

The problem (2) has a unique solution.

3. The Direct Result

We first define exactly what is meant by a dimensionally reduced solution to (2) . Let $\{\psi_j\}_{j=0}^{\infty} \subseteq H^1([-1,1])$ be a given sequence of linearly independent functions.

Definition

The dimensionally reduced solution u_N^h of order N is the projection of u^h onto the space

$$V_N^h = \{ \sum_{j=0}^{N} \psi_j(y/h) x_j \,|\, x_j \in \mathcal{D}(A^{1/2}), \quad j = 0,..,N\} \ .$$

The projection is with respect to the inner product $\mathcal{B}_h(u,v)$.

The main result of this section is the next theorem, which suggests a way of choosing $\{\psi_j\}_{j=0}^{\infty}$. It also gives an estimate of $|||u^h - u_N^h|||$ in terms of powers of h .

Let P denote the differential operator

$$b^{-1} \frac{d}{dy} a \frac{d}{dy}$$

In the notation of the previous section $P = -b^{-1} P_1(\frac{d}{dy})$. P is considered as an operator.

$$L^2([-1,1]) \supseteq \mathcal{D}(P) \to L^2([-1,1]) \ .$$

$\mathcal{N}(P^i)$ denotes the null space of the operator P^i , $0 \leq i$. $P^0 = I$ (identity). It is easy to see that $\mathcal{N}(P^i) \subseteq H^1([-1,1])$ for all i .

Theorem 3.1

There exists a sequence of linearly independent functions $\{\psi_j\}_{j=0}^{\infty}$, with

$$N(P^i) = \text{span } \{\psi_j\}_{j=0}^{2i-1} \qquad i \geq 1 ,$$

that has the following property:

For any integer $N \geq 0$ and for any given set of vectors $f, g \in \mathcal{D}((AM^{-1})^N)$ there exists a constant C_N (independent of h) such that

$$|||u^h - u^h_{2N}||| \leq C_N h^{2N+1/2} \qquad .$$

Remarks

The sequence $\{\psi_j\}_{j=0}^{\infty}$ depends only on the operator P. It is also clear that $\{\psi_j\}_{j=0}^{\infty}$ is not uniquely determined by Theorem 3.1. Any other sequence $\{\eta_j\}_{j=0}^{\infty}$, with span $\{\eta_j\}_{j=0}^{i}$ = span $\{\psi_j\}_{j=0}^{i}$ $\forall i$, could have been used.

The rest of this section is devoted to proving Theorem 3.1. To do so we need a couple of auxiliary results.

By changing variables to $[-1,1]$ and introducing $\tilde{u}^h(y) = u^h(y \cdot h)$ for $-1 \leq y \leq 1$ we transform (2) into the following equation for $\tilde{u}^h$:

$$(3) \qquad \begin{cases} \tilde{u}^h \in H^1([-1,1];H) \cap L^2([-1,1];\mathcal{D}(A^{1/2})) \quad , \\ \tilde{\mathcal{B}}_h(\tilde{u}^h, v) = \langle g, v(1)\rangle - \langle f, v(-1)\rangle \quad , \\ \forall v \in H^1([-1,1];H) \cap L^2([-1,1];\mathcal{D}(A^{1/2})) \quad . \end{cases}$$

Here the bilinear form $\tilde{\mathcal{B}}_h$ is given by

$$\tilde{\mathcal{B}}_h(u,v) = h^{-1} \int_{-1}^{1} a\langle M^{1/2} \frac{du}{dy}, M^{1/2} \frac{dv}{dy}\rangle \, dy + h \int_{-1}^{1} b \langle A^{1/2}u, A^{1/2}v\rangle \, dy \quad .$$

Now let us define the sequence $\{\psi_j^o\}_{j=0}^{\infty} \subseteq H^1([-1,1])$ by the following equations:

For any $v \in H^1([-1,1])$

$$(4^0) \qquad \int_{-1}^{1} a \frac{d\psi_0^o}{dy} \frac{dv}{dy} \, dy = 0 \quad ,$$

$$(4^1) \qquad \int_{-1}^{1} a \frac{d\psi_1^o}{dy} \frac{dv}{dy} \, dy + \int_{-1}^{1} b\psi_0^o v \, dy = v(1) \quad ,$$

and for $j \geq 2$:

$$(4^j) \qquad \int_{-1}^{1} a \frac{d\psi_j^o}{dy} \frac{dv}{dy} \, dy + \int_{-1}^{1} b\psi_{j-1}^o v \, dy = 0 \quad .$$

The sequence $\{\psi_j^1\}_{j=0}^{\infty} \subseteq H^1([-1,1])$ is defined by the same system of equations, the only difference being that in the right hand side of (4^1) $v(1)$ is replaced by $v(-1)$.

<u>Lemma 3.1</u>

Let j_o denote an integer ≥ 0 .

The equations (4^j) $0 \leq j \leq j_o$ determine the set $\{\psi_j^{\ell}\}_{j=0}^{j_o}$ uniquely up to a constant in $\psi_{j_o}^{\ell}$, $\ell = o,1$.

Proof

It is sufficient to prove the statement for $\ell = o$.

The case $j_o = 0$ is obvious. We proceed by induction. Hence assume that the equations (4^j) $0 \leq j \leq j_o - 1$ determine $\{\psi_j^o\}_{j=0}^{j_o-1}$ uniquely modulo a constant in $\psi_{j_o-1}^o$.

Consider the equations (4^j) $0 \leq j \leq j_o$. According to the induction hypothesis, $\{\psi_j^o\}_{j=0}^{j_o-1}$ is determined uniquely up to a constant in $\psi_{j_o-1}^o$. Choosing $v = 1$ in equation (4^{j_o}) we derive the value of $\int_{-1}^{1} b\, \psi_{j_o-1}^o\, dy$, which means that $\psi_{j_o-1}^o$ is completely determined. The equation (4^{j_o}) is now nothing but a Neuman problem for $-\frac{d}{dy}\, a\, \frac{d}{dy}$, and since $\int_{-1}^{1} b\, \psi_{j_o-1}^o\, v\, dy$ is equal to the right hand side of equation (4^{j_o}), for any constant v , this has a solution that is unique modulo a constant. This proves that $\psi_{j_o}^o$ is determined uniquely by up to a constant.

$\square$

Because of the way the two sequences $\{\psi_j^o\}_{j=0}^{\infty}$ and $\{\psi_j^1\}_{j=0}^{\infty}$ are constructed we also have

Lemma 3.2

For any $i \geq 0$

$$N(P^i) \not\subseteq \mathrm{span}\{\psi_j^o, \psi_j^1\}_{j=0}^{i} \not\subseteq N(P^{i+1}) \quad .$$

Proof

The lemma is clearly true, if for any $i \geq 0$ we can prove the more detailed statement:

$$N(P^i) \subsetneq \operatorname{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{i} \subsetneq N(P^{i+1}) \quad ,$$

and there exists a linear combination $\psi = \alpha\psi_{i+1}^o + \beta\psi_{i+1}^1$ such that

$$\operatorname{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{i} \oplus \operatorname{span}\{\psi\} = N(P^{i+1})$$

The validity of the above statement is easily checked for $i = 0$, and we proceed by induction. That is, we assume the statement to be true for $i = k \geq 0$. From the way ψ_{k+1}^o and ψ_{k+1}^1 are constructed it follows that

$$\psi_{k+1}^o,\ \psi_{k+1}^1 \in N(P^{k+2}) \setminus N(P^{k+1}) \quad ,$$

and because of the induction hypothesis we then get

$$N(P^{k+1}) \subsetneq \operatorname{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{k+1} \quad .$$

Now, adding the two vectors ψ_{k+1}^o and ψ_{k+1}^1 , we cannot increase the dimension by more than two. The fact that the codimension of $N(P^{k+1})$ in $N(P^{k+2})$ is 2 , together with the strict inclusion

$$\operatorname{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{k} \subsetneq N(P^{k+1}) \quad ,$$

hence shows that

$$\operatorname{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{k+1} \subsetneq N(P^{k+2}) \quad .$$

That means we have proven the first part of the extended statement for $i = k+1$.

Concerning the second part we consider the linear combination

$$\alpha\psi^o_{k+2} + \beta\psi^1_{k+2} \quad .$$

From the construction we know that

$$P(\alpha\psi^o_{k+2} + \beta\psi^1_{k+2}) = \alpha\psi^o_{k+1} + \beta\psi^1_{k+1} \quad .$$

Together with the induction hypothesis this tells that

$$\alpha\psi^o_{k+2} + \beta\psi^1_{k+2} \in N(P^{k+2}) \quad .$$

On the other hand, $\alpha\psi^o_{k+2} + \beta\psi^1_{k+2}$ cannot be an element of $\mathrm{span}\{\psi^o_j,\psi^1_j\}^{k+1}_{j=0}$. If so, we would have by application of the operator P that

$$\alpha\psi^o_{k+1} + \beta\psi^1_{k+1} \in \mathrm{span}\{\psi^o_j,\psi^1_j\}^{k}_{j=0} \quad ,$$

which contradicts the induction hypothesis.

This finally proves that $\alpha\psi^o_{k+2} + \beta\psi^1_{k+2}$ together with $\mathrm{span}\{\psi^o_j,\psi^1_j\}^{k+1}_{j=0}$ spans all of $N(P^{k+2})$.

Thus the second part has been established for $i = k+1$.

$\square$

In the proof of Theorem 3.1 we shall also use the following density result.

Lemma 3.3

The set V defined as

$$V = \{\sum_{j=0}^{J} v_j\, x_j \mid J \in \mathbb{N},\ x_j \in \mathcal{D}(A^{1/2}) \text{ and } v_j \in H^1([-1,1]) \text{ for } 0 \leq j \leq J\}$$

is dense in $H^1([-1,1];H) \cap L^2([-1,1];\mathcal{D}(A^{1/2}))$.

Proof

The lemma is clearly proven if one shows

(i) V is dense in $H^1([-1,1];\mathcal{D}(A^{1/2}))$;

(ii) $H^1([-1,1];\mathcal{D}(A^{1/2}))$ is dense in $H^1([-1,1];H) \cap L^2([-1,1];\mathcal{D}(A^{1/2}))$.

Let us start with (i). Assume $u \in V^{\perp}$ ($\perp$ in $H^1([-1,1];\mathcal{D}(A^{1/2}))$), that is,

$$\int_{-1}^{1} \langle A^{1/2} \frac{du}{dy}, A^{1/2}x\rangle \frac{dv}{dy}\, dy + \int_{-1}^{1} \langle A^{1/2}u, A^{1/2}x\rangle\, v\, dy = 0$$

$$\forall v \in H^1([-1,1]), \quad x \in \mathcal{D}(A^{1/2}) \quad .$$

Since A is closed, this yields

$$\int_{-1}^{1} \frac{d}{dy} \langle A^{1/2}u, A^{1/2}x\rangle \frac{dv}{dy}\, dy + \int_{-1}^{1} \langle A^{1/2}u, A^{1/2}x\rangle\, v\, dy = 0$$

$\forall v \in H^1([-1,1]),\ \ x \in \mathcal{D}(A^{1/2})$, and hence

$$\langle A^{1/2}u, A^{1/2}x\rangle = 0 \quad \forall x \in \mathcal{D}(A^{1/2}) \quad ,$$

or

$$\langle u, Ax \rangle = 0 \qquad \forall x \in \mathcal{D}(A) \quad .$$

Because of the fact that A is invertible, we know that $A(\mathcal{D}(A)) = H$. From the previous equation we thus get

$$u = 0 \quad .$$

In summary, we have now proven that $V^{\perp} = 0$. This immediately yields (i) .

To prove (ii), let dE_λ be the spectral measure of $A^{1/2}$ (cf. [7]). If $u \in H^1([-1,1];H) \cap L^2([1,1];\mathcal{D}(A^{1/2}))$, then u can be written as

$$u(y) = \int_{\sigma(A^{1/2})} dE_\lambda u(y) \quad ,$$

where

$$\int_{\sigma(A^{1/2})} \lambda^2 d||E_\lambda u(y)||^2 < \infty \quad .$$

Now define

$$u_k(y) = \int_{\sigma(A^{1/2}) \cap [0,k]} dE_\lambda u(y) \quad .$$

Obviously $u_k \in H^1([-1,1];\mathcal{D}(A^{1/2}))$, and

$$\int_{-1}^{1} || \frac{d}{dy} (u-u_k)||^2 dy =$$

$$\int_{-1}^{1} \int_{\sigma(A^{1/2}) \cap]k,\infty[} d||E_\lambda \frac{d}{dy} u||^2 \, dy \to 0$$

for $k \to \infty$, because of Lebesque's monotone convergence theorem. In the same fashion

$$\int_{-1}^{1} ||A^{1/2}(u-u_k)||^2 dy =$$

$$\int_{-1}^{1} \int_{\sigma(A^{1/2}) \cap]k,\infty[} \lambda^2 d||E_\lambda u||^2 dy \to 0$$

for $k \to \infty$. That is, we have proven

$$|||u-u_k||| \to 0 \quad \text{for} \quad k \to \infty \quad ,$$

which immediately gives (ii).

$\square$

Finally we are now ready for the

<u>Proof of Theorem 3.1</u>

Choose $\{\psi_j\}_{j=0}^{\infty}$ such that

$$\text{span}\{\psi_j\}_{j=0}^{2i-1} = N(P^i) \qquad \forall i \geq 1 \quad ,$$

and $$\text{span}\{\psi_j\}_{j=0}^{2i} = \text{span}\{\psi_j^o, \psi_j^1\}_{j=0}^{i} \qquad \forall i \geq 0 \quad .$$

This is possible because of Lemma 3.2. The ψ_j's chosen this way obviously have the first property stated in Theorem 3.1 .

For any $N \geq 0$ and any pair $f,g \in \mathcal{D}((AM^{-1})^N)$ let

$$S_N^h = \sum_{j=0}^{N} h^{-1+2j}(\psi_j^o(y/h)M^{-1}(AM^{-1})^{j-1}g - \psi_j^1(y/h)M^{-1}(AM^{-1})^{j-1}f) \quad .$$

It is clear that $S_N^h \in V_{2N}^h$. Because of Proposition 2.1, it follows that there exists a constant C (independent of N and h) such that

$$|||u^h - u_{2N}^h||| \leq C \inf_{v \in V_{2N}^h} |||u^h - v||| \leq C|||u^h - S_N^h||| \quad .$$

We now estimate

$$|||u^h - S_N^h||| \quad .$$

By a change of variables and introduction of the bilinear form $\tilde{\mathcal{B}}_h$ it follows that

$$|||u^h - S_N^h||| \leq C[\tilde{\mathcal{B}}_h(\tilde{u}^h - \tilde{S}_N^h, \tilde{u}^h - \tilde{S}_N^h)]^{1/2} \quad ,$$

with $\tilde{S}_N^h = \sum_{j=0}^{N} h^{-1+2j}(\psi_j^o(y)M^{-1}(AM^{-1})^{j-1}g - \psi_j^1(y)M^{-1}(AM^{-1})^{j-1}f)$ (and C independent of N and h).

Let us now consider

$$\tilde{\mathcal{B}}_h(\tilde{u}^h - \tilde{S}_N^h, v)$$

where v has the form $w(y)\cdot x$, $x \in \mathcal{D}(A^{1/2})$ and $w \in H^1([-1,1])$. We know that

$$\tilde{\mathcal{B}}_h(\tilde{u}^h - \tilde{S}_N^h, v) = w(1)\ \langle g,x\rangle - w(-1)\ \langle f,x\rangle - \tilde{\mathcal{B}}_h(\tilde{S}_N^h, v) \quad .$$

Concerning the last term we have

$$\tilde{B}_h(\tilde{S}_N^h,v) = \sum_{j=0}^{N} h^{-1+2j}\tilde{B}_h(\psi_j^o M^{-1}(AM^{-1})^{j-1}g,v) -$$

$$- \sum_{j=0}^{N} h^{-1+2j}\tilde{B}_h(\psi_j^1 M^{-1}(AM^{-1})^{j-1}f,v) \quad .$$

Now because of the properties of the sequence $\{\psi_j^o\}_{j=0}^{\infty}$

$$\sum_{j=0}^{N} h^{-1+2j}\tilde{B}_h(\psi_j^o M^{-1}(AM^{-1})^{j-1}g,v) =$$

$$\sum_{j=0}^{N} h^{-2+2j}\int_{-1}^{1} a\,\frac{d\psi_j^o}{dy}\,\frac{dw}{dy}\,dy < (AM^{-1})^{j-1}g,x > +$$

$$+ \sum_{j=0}^{N} h^{2j}\int_{-1}^{1} b\psi_j^o w dy < (AM^{-1})^{j}g,x > =$$

$$w(1) <g,x> - h^{2N}\int_{-1}^{1} a\,\frac{d\psi_{N+1}^o}{dy}\,\frac{dw}{dy}\,dy < (AM^{-1})^{N}g,x > \quad .$$

Similarly

$$\sum_{j=0}^{N} h^{-1+2j}\tilde{B}_h(\psi_j^1 M^{-1}(AM^{-1})^{j-1}f,v) =$$

$$w(-1) <f,x> - h^{2N}\int_{-1}^{1} a\,\frac{d\psi_{N+1}^1}{dy}\,\frac{dw}{dy}\,dy < (AM^{-1})^{N}f,x > \quad ,$$

so that altogether

$$\tilde{B}_h(\tilde{u}^h-\tilde{S}_N^h,v) = h^{2N}\int_{-1}^{1} a < \frac{dr_N}{dy} , \frac{dv}{dy} > dy$$

for any v of the form $w(y)\cdot x$, $x \in \mathcal{D}(A^{1/2})$ and $w \in H^1([-1,1])$. r_N is given by

$$r_N = \psi^o_{N+1}(AM^{-1})^N g - \psi^1_{N+1}(AM^{-1})^N f \quad .$$

Now, because of continuity and Lemma 3.3, we get that

$$\tilde{\mathcal{B}}_h(\tilde{u}^h - \tilde{S}^h_N, v) = h^{2N} \int_{-1}^{1} a < \frac{dr_N}{dy}, \frac{dv}{dy} > dy$$

$$\forall\, v \in H^1([-1,1];H) \cap L^2([-1,1];\mathcal{D}(A^{1/2})) \quad .$$

Using Schwarz's inequality, we see that the right hand side is bounded by

$$C_N\, h^{2N+1/2}[\tilde{\mathcal{B}}_h(v,v)]^{1/2} \quad (C_N \text{ independent of } h).$$

As a consequence of this it follows that

$$[\tilde{\mathcal{B}}_h(\tilde{u}^h - \tilde{S}^h_N,\ \tilde{u}^h - \tilde{S}^h_N)]^{1/2} \leq C_N\, h^{2N+1/2} \quad ,$$

and hence $|||u^h - S^h_N||| \leq C_N\, h^{2N+1/2}$.

$\square$

4. The Inverse Result

In Theorem 3.1 we examined a particular choice of the functions ψ_j . The approximation error was of order $h^{2N+1/2}$ using the $2N+1$ functions $\{\psi_j\}_{j=0}^{2N}$.

The goal of this section is to prove that the previous choice of functions was by no means arbitrary. That sequence, or any other sequence $\{\eta_j\}_{j=0}^{\infty}$ with span $\{\eta_j\}_{j=0}^{2i}$ = span $\{\psi_j\}_{j=0}^{2i}$ for every i , is the only one that gives this order of approximation.

We formulate this as

<u>Theorem 4.1</u>

Let N and K be two non-negative integers and f and g two linearly independent elements of H . Let $\{\psi_j\}_{j=0}^{\infty}$ be the sequence introduced in Theorem 3.1 and u^h the solution to (2) of section 2.

If $\{\phi_j\}_{j=0}^{K}$ is a set of elements of $H^1([-1,1])$ with the property that

$$\inf_{v \in W_K^h} |||u^h - v||| = o(h^{\max\{2N-3/2,-1/2\}})$$

where W_K^h denotes the set

$$\{ \sum_{j=0}^{K} \phi_j(y/h) x_j \mid x_j \in \mathcal{D}(A^{1/2}) \quad 0 \le j \le K \} ,$$

then

$$\mathrm{span}\{\psi_j\}_{j=0}^{2N} \subseteq \mathrm{span}\{\phi_j\}_{j=0}^{K} .$$

Theorem 4.1 is actually a little stronger than just an inverse of Theorem 3.1. Let N be ≥ 1 . Theorem 3.1 then says that with the $2N+1$ functions $\{\psi_j\}_{j=0}^{2N}$ we can obtain an error of order $h^{2N+1/2}$. But Theorem 4.1 tells us that even if we are satisfied with an error of order $o(h^{2N-3/2})$, we still have to use all the functions $\{\psi_j\}_{j=0}^{2N}$.

Proof

We can, without loss of generality, assume that $f, g \in \mathcal{D}((AM^{-1})^N)$. Otherwise we replace u^h, f and g by $u^{h*} = (MA^{-1})^N u^h$, $f^* = (MA^{-1})^N f$ and $g^* = (MA^{-1})^N g$, which obviously satisfy the assumptions of the theorem.

Define S_N^h , $\tilde{S}_N^h$ as in the proof of Theorem 3.1, i.e.,

$$S_N^h = \sum_{j=0}^{N} h^{-1+2j} \left(\psi_j^o(y/h)M^{-1}(AM^{-1})^{j-1}g - \psi_j^1(y/h)M^{-1}(AM^{-1})^{j-1}f\right)$$

and $\tilde{S}_N^h = S_N^h(y\cdot h)$.

Then because of Theorem 3.1 and the assumption of this theorem we see that $\exists\, v^h \in W_K^h$ such that

$$|||S_N^h - v^h||| = o(h^{\max\{2N-3/2,\ -1/2\}}) \quad .$$

By a change of variables, this yields

$$\text{(i)} \qquad \int_{-1}^{1} ||\frac{d}{dy}(\tilde{S}_N^h - \tilde{v}^h)||^2\, dy = o(h^{\max\{4N-2,\ 0\}}) \quad ,$$

$$\text{(ii)} \qquad \int_{-1}^{1} ||A^{1/2}(\tilde{S}_N^h - \tilde{v}^h)||^2 dy = o(h^{\max\{4N-4,-2\}}) \quad .$$

The function $\tilde{v}^h$ has the form

$$\sum_{j=0}^{K} \phi_j(y)v_j^h \quad , \quad \text{with} \quad v_j^h \in \mathcal{D}(A^{1/2}) \qquad \text{for} \quad 0 \leq j \leq K \ .$$

Now, multiplying (ii) through by h^2 , we get because of the form of $\tilde{S}_N^h$ and $\tilde{v}^h$ that

$$\int_{-1}^{1} ||\psi_0^o(y)A^{-1/2}g - \psi_0^1(y)A^{-1/2}f - h\sum_{j=0}^{K}\phi_j(y)A^{1/2}v_j^h||^2 dy \to 0$$

for $h \to 0$. Since for a fixed K the set

$$\{\sum_{j=0}^{K}\phi_j(y)x_j \mid x_j \in H\}$$

is closed in $L^2([-1,1]; H)$, we get

$$\psi_0^o(y)A^{-1/2}g - \psi_0^1(y)A^{-1/2}f = \sum_{j=0}^{K}\phi_j(y)x_j \quad .$$

The fact that f and g are linearly independent implies that so are $A^{-1/2}f$ and $A^{-1/2}g$. Hence

$$\text{span}\,\{\psi_0^o, \psi_0^1\} \subseteq \text{span}\,\{\phi_j\}_{j=0}^{K} \quad .$$

This proves Theorem 4.1 for $N = 0$. If $N \geq 1$ we are not yet finished. In this case we proceed by induction, i.e., we assume it has been proven that

$$\text{span}\,\{\psi_j^o, \psi_j^1\}_{j=0}^{m} \subseteq \text{span}\,\{\phi_j\}_{j=0}^{K}$$

for $0 \leq m \leq N-1$.

Rearranging (i) and dividing through by h^{4m+2} we get

$$\int_{-1}^{1} ||\frac{d\psi_{m+1}^o}{dy} M^{-1}(AM^{-1})^m g - \frac{d\psi_{m+1}^1}{dy} M^{-1}(AM^{-1})^m f +$$

$$+ \sum_{j=0}^{m} h^{2(j-m-1)}\left(\frac{d\psi_j^o}{dy} M^{-1}(AM^{-1})^{j-1}g - \frac{d\psi_j^1}{dy} M^{-1}(AM^{-1})^{j-1}f\right) -$$

$$- h^{-2m-1} \sum_{j=0}^{K} \frac{d\phi_j}{dy} v_j^h||^2 dy \to 0 \quad \text{for} \quad h \to 0 \quad .$$

(Here we use that $4m+2 \leq 4N-2$).

Because of the induction hypothesis we know that

$$\sum_{j=0}^{m} h^{2(j-m-1)} \left(\frac{d\psi_j^o}{dy} M^{-1}(AM^{-1})^{j-1}g - \frac{d\psi_j^1}{dy} M^{-1}(AM^{-1})^{j-1}f\right) = \sum_{j=0}^{K} \frac{d\phi_j}{dy} x_j^h \ ,$$

and now, using that for a fixed K the set

$$\{\sum_{j=0}^{K} \frac{d\phi_j}{dy} x_j \mid x_j \in H\}$$

is closed in $L^2([1,1]);H)$, *we conclude that*

$$\frac{d\psi_{m+1}^o}{dy} M^{-1}(AM^{-1})^m g - \frac{d\psi_{m+1}^1}{dy} M^{-1}(AM^{-1})^m f = \sum_{j=0}^{K} \frac{d\phi_j}{dy} x_j \quad .$$

The fact that f and g are linearly independent implies that so are $M^{-1}(AM^{-1})^m g$ and $M^{-1}(AM^{-1})^m f$. Hence

$$\frac{d\psi_{m+1}^o}{dy} , \frac{d\psi_{m+1}^1}{dy} \in \text{span}\{\frac{d\phi_j}{dy}\}_{j=0}^{K} \quad .$$

From the way ψ_0^o and ψ_0^1 are constructed it is easily seen that

$$\text{span}\{\psi_0^o,\psi_0^1\} = \{\text{constant functions}\} \quad ,$$

and this together with the induction hypothesis allows us to conclude that

$$\text{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{m+1} \subseteq \text{span}\{\phi_j\}_{j=0}^{K} \quad .$$

The induction proof is now finished, and finally we get because of the definition of $\{\psi_j\}_{j=0}^{2N}$:

$$\mathrm{span}\{\psi_j\}_{j=0}^{2N} = \mathrm{span}\{\psi_j^o,\psi_j^1\}_{j=0}^{N} \subseteq \mathrm{span}\{\phi_j\}_{j=0}^{K} \quad .$$

$\square$

By a slight variation of the preceding arguments we could prove the following version of Theorem 4.1, for the case where f and g are not linearly independent, e.g., $f = \alpha\cdot g$.

Theorem 4.2

Let N and K be non-negative integers. Let $f = \alpha\cdot g$, with $g\in H\setminus\{0\}$.

Let $\{\psi_j\}_{j=0}^{\infty}$ be the sequence introduced in Theorem 3.1.

If $\{\phi_j\}_{j=0}^{K}$ is a set of elements of $H^1([-1,1])$ with the property that

$$\inf_{v\in W_K^h} |||u^h-v||| = O(h^{2N-1/2}) \quad ,$$

then

$$\mathrm{span}\{\psi_j^o - \alpha\psi_j^1\}_{j=0}^{N} \subseteq \mathrm{span}\{\phi_j\}_{j=0}^{K} \quad .$$

5. Numerical Examples

Consider the problem

$$\begin{cases} \operatorname{div}(a \operatorname{grad} u^h) = 0 \quad \text{in} \quad]0,1[\times]-h,h[\quad , \\ \qquad a \dfrac{\partial}{\partial y} u^h = g(x) \quad \text{for} \quad y = h \ , \\ \qquad a \dfrac{\partial}{\partial y} u^h = -g(x) \quad \text{for} \quad y = -h \ , \\ \qquad\quad u^h = 0 \quad \text{for} \quad x = 0 \quad \text{and} \quad x = 1 \ , \end{cases}$$

$$\text{with} \quad a(y) = \begin{cases} a_+ & \text{for} \quad y \geq 0 \\ a_\div & \text{for} \quad y < 0 \end{cases} \quad .$$

(a_+ and $a_\div$ are two positive constants). This problem clearly falls within the framework of our model problem. Simply choose

$$b(y) = a(y) = \begin{cases} a_+ & \text{for} \quad y \geq 0 \\ a_\div & \text{for} \quad y < 0 \end{cases} \quad ,$$

$$c(x) = 1 \quad \text{and}$$

$$A = -\left(\frac{\partial}{\partial x}\right)^2 \quad \text{with} \quad \mathcal{D}(A) = H^2([0,1]) \cap \overset{\circ}{H}{}^1([0,1])$$

$$\text{and} \quad \mathcal{H} = L^2([0,1]) \quad .$$

The operator P in this case is given by

$$P = a^{-1} \frac{d}{dy} a \frac{d}{dy} \quad .$$

Define $\{\phi_j\}_{j=0}^{\infty} \subseteq H^1([-1,1])$ as follows

$$\phi_0 = 1 \ , \quad \phi_1(y) = a^{-1} y \ ,$$

$$\phi_{2j}(y) = \int_{-1}^{y} \ell_{2j-1}(t)dt \quad \text{for} \quad j \geq 1$$

and

$$\phi_{2j+1}(y) = a^{-1} \int_{-1}^{y} \ell_{2j}(t)dt \quad \text{for} \quad j \geq 1 .$$

Here ℓ_k denotes the Legendre polynomial of degree k .

It is not difficult to see that with this definition

$$\text{span}\{\phi_j\}_{j=0}^{i} = \text{span}\{\psi_j\}_{j=0}^{i} \quad \forall i \geq 0 \quad ,$$

where $\{\psi_j\}_{j=0}^{\infty}$ is the sequence introduced in Theorem 3.1.

As before u_N^h denotes the dimensionally reduced solution of order N . u_N^h has the form

$$u_N^h = \sum_{j=0}^{N} \phi_j(y/h)u_j(x) \quad ,$$

with $u_j \in \overset{\circ}{H}{}^1([0,1])$. The vector $U = (u_j)_{j=0}^{N}$ is the solution to a two-point elliptic boundary value problem

$$-h \underline{\underline{K}} \left(\frac{d}{dx}\right)^2 U + 1/h \underline{\underline{L}} U = F$$

$$U(0) = U(1) = 0 .$$

The matrix $\underline{\underline{L}}$ is diagonal, and the matrix $\underline{\underline{K}}$ has a band structure. Both $\underline{\underline{L}}$ and $\underline{\underline{K}}$ are independent of h . We solve this problem numerically by expanding U in its Fourier series, only maintaining a finite number of terms. Since we are interested in studying the error introduced by the dimensional reduction, we maintain a very high number of terms. The graphs shown here were computed using 400 Fourier coefficients. This ensures that the error introduced by

discretization can be neglected compared to the error introduced by dimensional reduction.

Let us start with the case $a_+ = a_\div = 1$ and $g(x) = \pi/4$. Figures 1, 2 and 3 show the energy error as a function of h by dimensional reduction of order 0, 2 and 4 respectively. (Note that the functions $\{\phi_j\}_{j=0}^{\infty}$ form a basis for the polynomials in this case.) Using interpolation by the K-method (cf.[3]) we know that $g(x) \in (H, \mathcal{D}(A))_{1/4,\infty}$. An application of Theorem 3.1 hence gives the following conclusions.

i) The energy error is of order h with dimensional reduction based on ϕ_0 .

ii) The energy error is of order h^2 with dimensional reduction based on ϕ_0, ϕ_1 and ϕ_2 .

iii) The energy error is of order h^2 with dimensional reduction based on $\{\phi_j,\ 0 \le j \le 4\}$.

Figures 1, 2 and 3 illustrate the sharpness of the theoretical results. Comparing figures 2 and 3 we see that

> The requirement $f, g \in \mathcal{D}((AM^{-1})^N)$ in Theorem 3.1 is essential. If f and g are not sufficiently smooth, higher order dimensionally reduced models will not improve the asymptotic order of approximation as $h \to 0$.

Now consider the case where $a_+ \neq a_\ddagger$. In our computations $a_+ = 1$, $a_\ddagger = 2$ and again $g(x) = \pi/4$. Figure 4 compares two different dimensionally reduced solutions. For one of the dimensionally reduced solutions the polynomials of degree ≤ 2 have been chosen as basis functions in the y-direction. For the other dimensionally reduced solution the "special" functions ϕ_0, ϕ_1

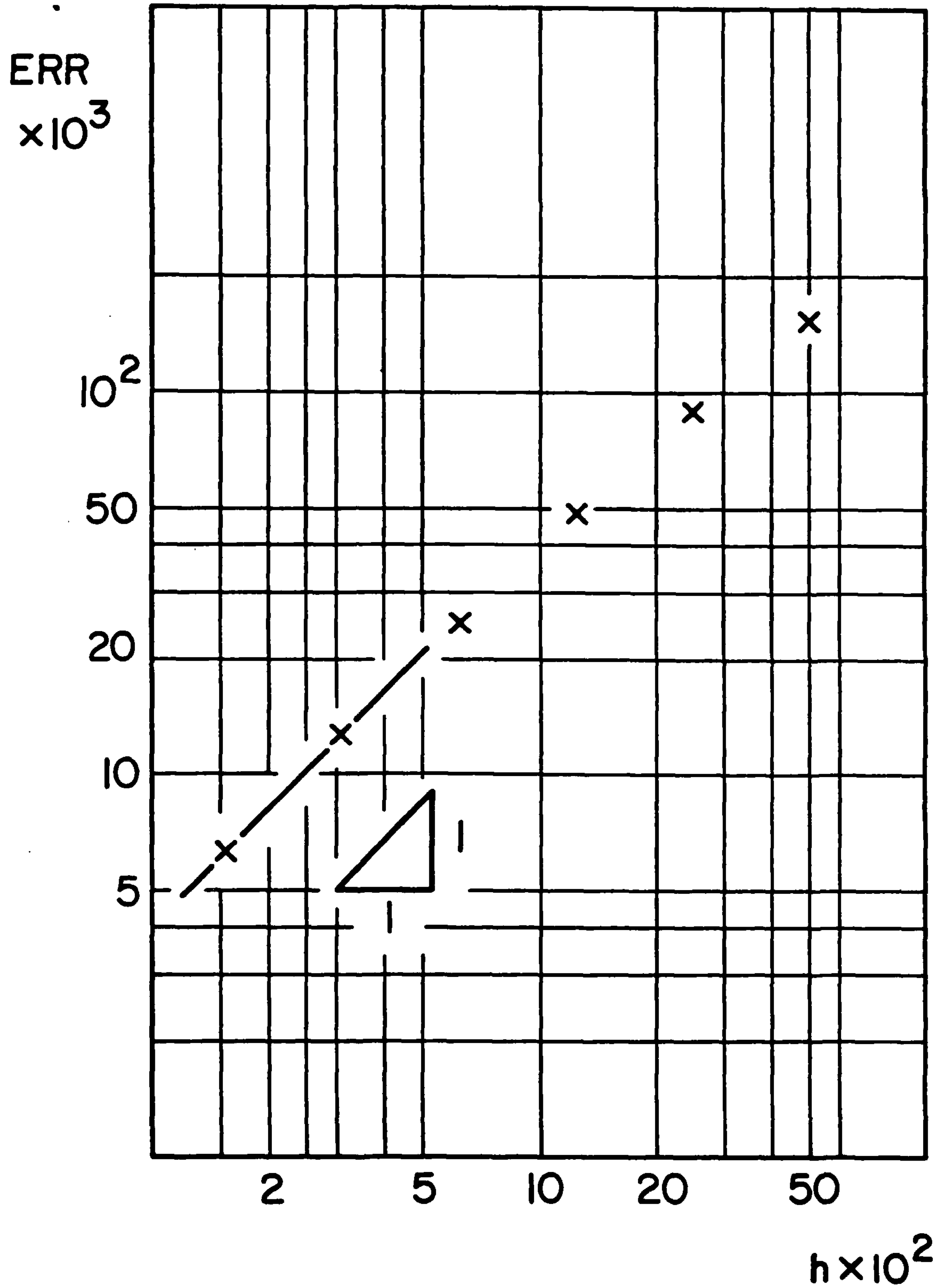

Fig. 1: Energy error $x10^3$ as a function of h , using polynomials of degree = 0 .

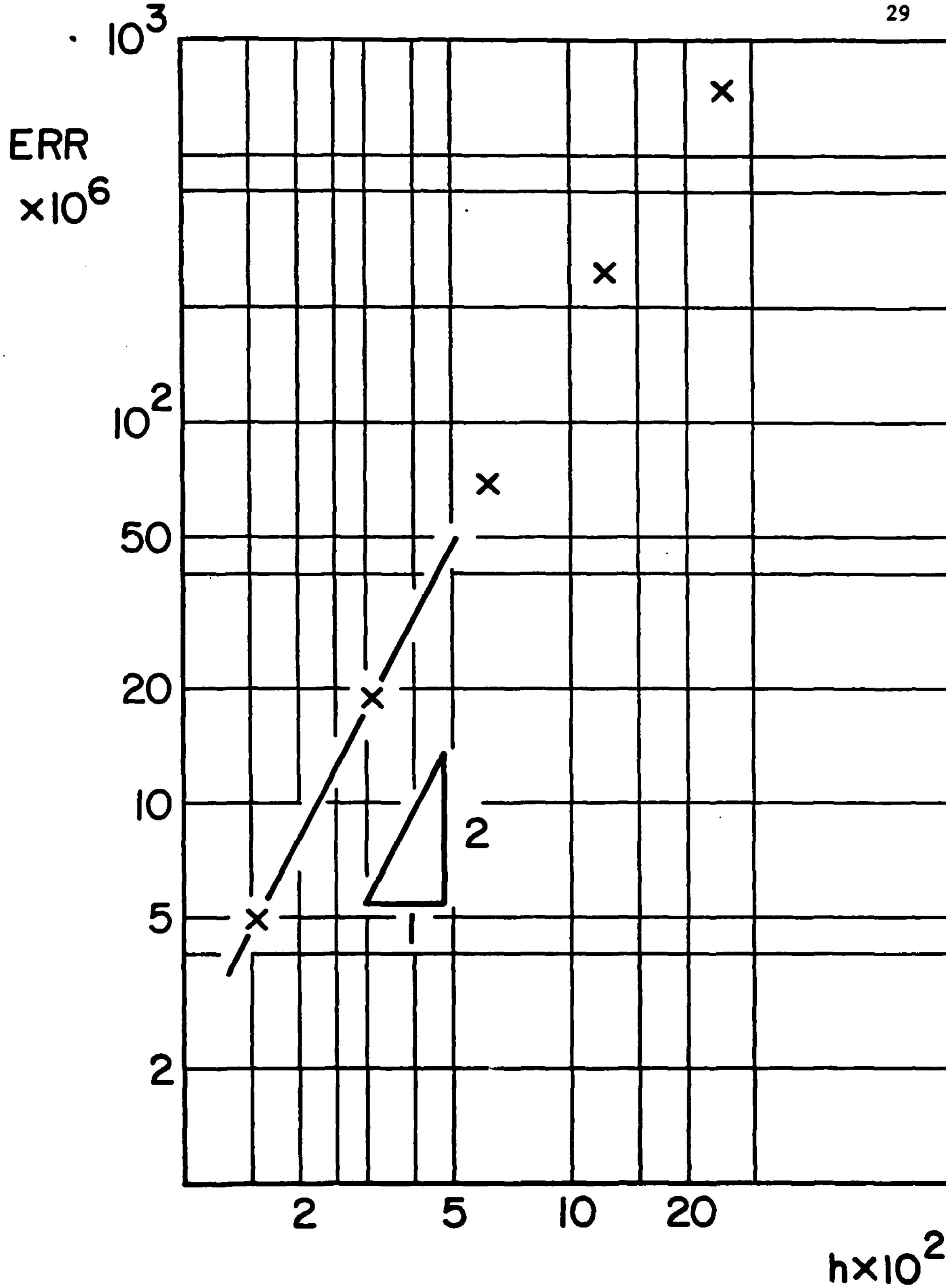

Fig. 2: Energy error x10^6 as a function of h , using polynomials of degree ≤ 2 .

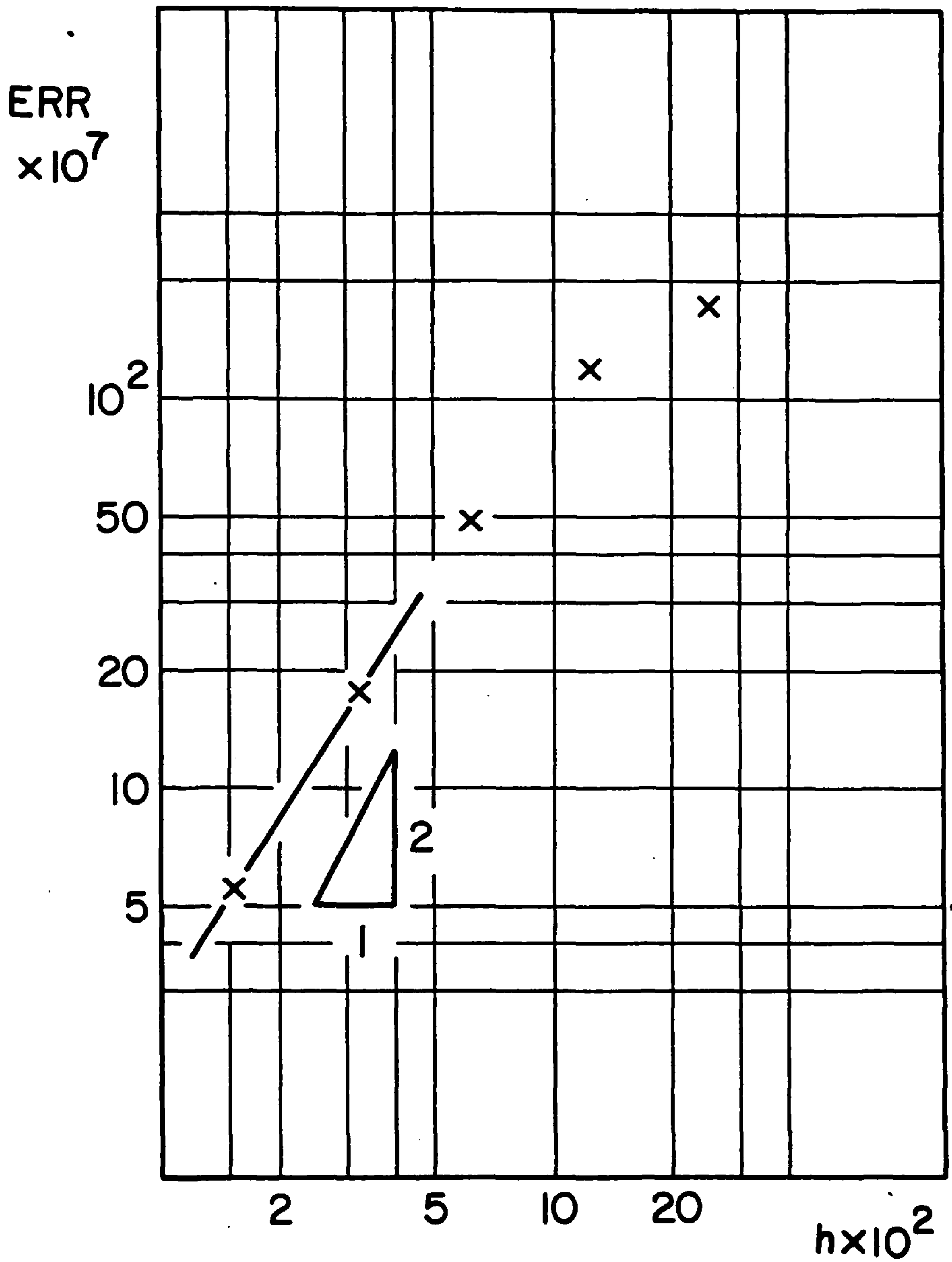

Fig. 3: Energy error $x10^7$ as a function of h , using polynomials of degree ≤ 4 .

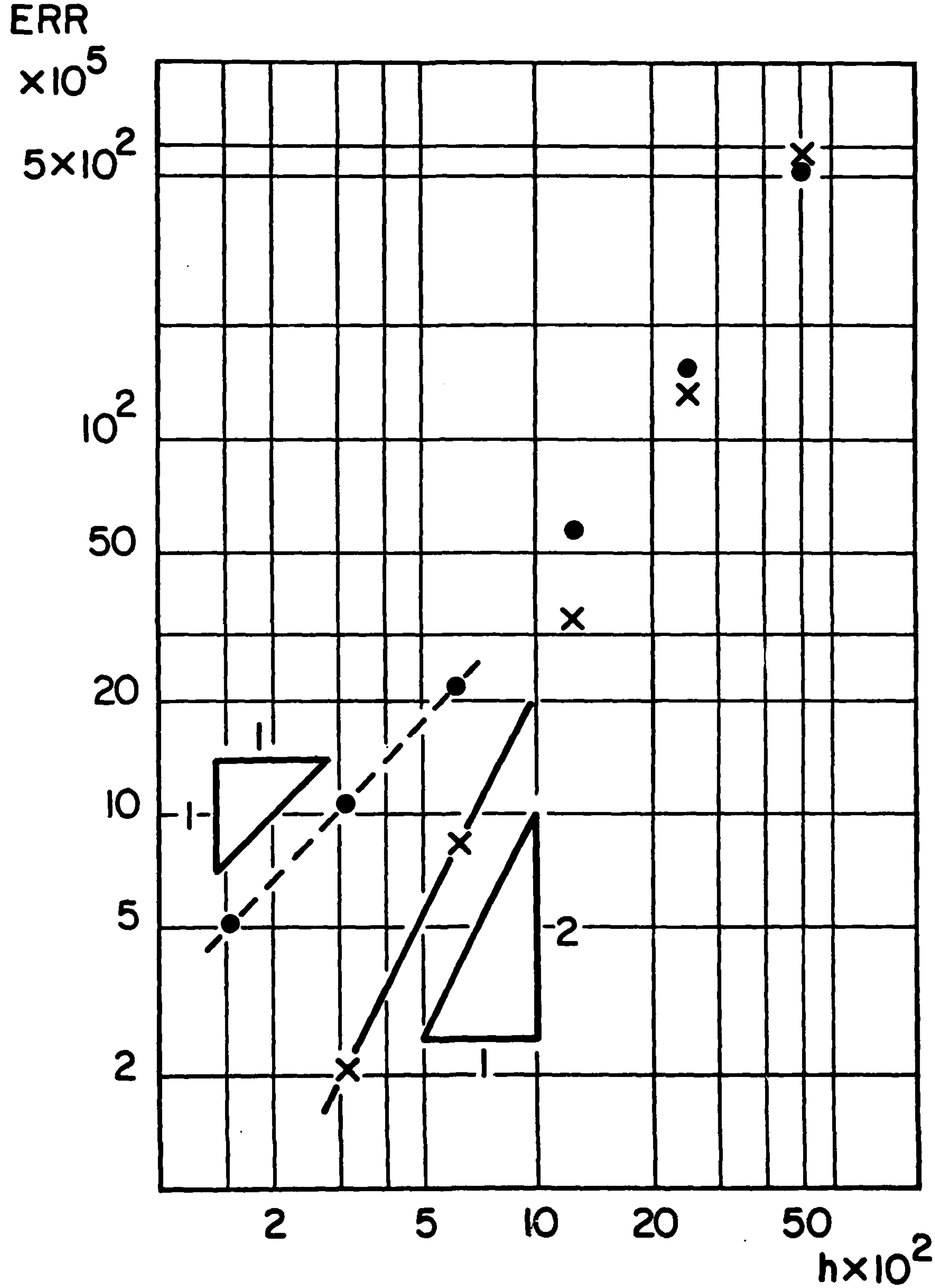

Fig. 4: Energy error $\times 10^5$ as a function of h,

—●— ●— using polynomials of degree ≤ 2,

-X——X- using the "special" functions ϕ_0, ϕ_1 and ϕ_2 .

and ϕ_2 , introduced earlier in this section, have been used. (Note that the "special" functions are piecewise polynomials in this case.)

Applying Theorem 3.1 we expect that

> The energy error will be of order h^2 with dimensional reduction based on the functions ϕ_0, ϕ_1 and ϕ_2 .

Since the "special" function ϕ_0 is the constant $= 1$, which of course is a polynomial of degree ≤ 2 , we also expect that

> The energy error will be of order h with dimensional reduction based on the polynomials of degree ≤ 2 .

From Fig. 4 it is again evident that there is a very good agreement between the theory and the computational results. Specifically it is seen, by comparing figures 2 and 4, that

> If the dimensional reduction is based on the "special" functions $\{\phi_j\}_{j=0}^{\infty}$, then the asymptotic behaviour of the energy error is independent of the regularity of the solution u^h across the line $y = 0$.

A feature very similar to this is well known for optimally constructed Finite Element meshes.

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